# Companion Volume II

## Dimensional Correlation

Non-Normative Companion to the LMR Predynamical Codex
Length–Mass Reduction (LMR) Theory

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v0.2 — Notation Lock Applied

*Changes from v0.1.*

This revision applies the locked notation convention established in the LMR Tier 3 Glossary. Every Tier 3 reduced quantity now carries an SI left-subscript (${}_{\mathrm{SI}}M'$, ${}_{\mathrm{SI}}E'$, ${}_{\mathrm{SI}}P'$, ${}_{\mathrm{SI}}F'$, ${}_{\mathrm{SI}}q'$, ${}_{\mathrm{SI}}G'$, ${}_{\mathrm{SI}}\varepsilon'_0$, ${}_{\mathrm{SI}}\mu'_0$, ${}_{\mathrm{SI}}T'_{\mu\nu}$, ${}_{\mathrm{SI}}V'$) to typographically distinguish it from any bare-primed quantity that may appear in Arc I (Tier 1) under the same letter. The Preface and Definition 1.2 have been updated to lock the convention; the verbal disambiguation paragraph at §10.6 has been updated to acknowledge the new notation. No algebraic, derivational, or empirical content has been changed. All reductions, identities, and reduced equations are identical to v0.1 in algebraic substance.

# Reading Guide: Structural Grammar, Overlay, and Dimensional Correlation

## Purpose of This Guide

The LMR framework is organized into three formally distinct tiers. These tiers are independent in role, scope, and authority. Confusion between tiers leads to category error.

This guide specifies:

- What each tier does,
- What each tier does not do,
- How information flows between tiers,
- What is prohibited from flowing between tiers.

No physical claim should be interpreted outside its proper tier.

## Tier 1 — Structural Grammar (Papers I–V)

Tier 1 defines the predynamical structural grammar of LMR. It establishes:

- Corridor primitives,
- Admissibility conditions,
- Closure structure,
- Torsion engagement,
- Class identity.

Tier 1 is pre-dynamical, time-independent, non-probabilistic, non-statistical, and ontologically declarative.

Tier 1 does *not*: introduce evolution equations, hazard functions, transport equations, stochastic mechanisms; perform dimensional reduction on SI; or appeal to empirical fitting. Structural grammar determines admissibility. It does not determine dynamics.

## Tier 2 — Overlay Layer (Companion Volume I)

Tier 2 introduces declared representational overlays. An overlay is a formally stated mapping from structural scale parameters to observer-side equations.

Tier 2 operates only on structural quantities already defined in Tier 1; introduces Model Rules (M#), Dynamical Postulates (D#), and Structural Constraints (SC#); and derives observer-side equation families under explicit rule declarations.

Tier 2 does *not* modify structural grammar, introduce new structural primitives, claim that any admitted equation is structurally inevitable, or promote representational equations to ontological status. Time appears in Tier 2 only through declared update postulates; no time parameter exists in Tier 1. Multiple distinct equation families may arise from the same structural cadence scale depending on declared update symmetry. This non-uniqueness is intentional.

### Tier 3 — Dimensional Correlation Layer (this volume)

Tier 3 performs $\ell_m$-reduction on standard SI equations. Its purpose is dimensional translation and representational alignment.

Tier 3 eliminates explicit kilogram dependence; introduces the bridge quantity $\ell_m = h/c$; rewrites SI equations in kg-free form; preserves algebraic structure exactly; and introduces no new physics.

Tier 3 does *not* assert structural primitives, corridor operators, or admissibility claims; introduce dynamical postulates; derive new empirical predictions; or modify established theory. Tier 3 performs representational alignment only. Interpretive conclusions cannot arise from dimensional reduction alone.

### Inter-Tier Discipline

The tiers are orthogonal.

- Tier 1 does not imply Tier 2.
- Tier 2 does not modify Tier 1.
- Tier 3 does not justify Tier 1.
- Tier 3 does not derive Tier 2.

**Permitted flow.** Tier 3 may compare reduced SI expressions with Tier 1 quantities only through explicit declaration. Tier 1 supplies dimensional primitives to Tier 3. Tier 2 may be dimensionally expressed through Tier 3.

**Prohibited flow.** No overlay equation may be used to infer structural ontology. No dimensional re-expression may be used to assert structural necessity. No SI algebra may be used to define structural primitives.

### How to Read This Work

When encountering an equation, ask:

1. Is this a structural statement? (Tier 1)
2. Is this an overlay postulate? (Tier 2)
3. Is this a dimensional re-expression of SI? (Tier 3)

Interpret the equation only within its tier. Cross-tier inference without explicit declaration is invalid.

### Summary

LMR consists of: a predynamical structural grammar; a declared overlay layer admitting representational evolution equations; and a dimensional correlation layer eliminating kilogram dependence. These tiers must remain formally separated for the framework to remain coherent.

## Preface

This volume presents a systematic elimination of kilogram dependence from standard SI equations. It introduces no new physical assumptions, modifies no empirical predictions, and proposes no reinterpretation of established theory.

The procedure rests on the dimensional bridge

$$\ell_m := \frac{h}{c}.$$

The quantity $\ell_m$ carries units of kg·m and permits consistent removal of the kilogram from any SI expression in which it appears. The resulting equations are algebraically equivalent to their SI forms. Only their dimensional representation changes.

### Notation Convention (v0.2)

This revision locks a three-state notational convention that disambiguates Tier 3 reduced quantities from any bare-primed forms appearing elsewhere in the LMR program:

1. **No prime** — the quantity is SI-unchanged (its dimensions contain no kilogram) and identical across Tier 1, Tier 2, and Tier 3. Examples: $c$, $v$, $\lambda$, $t$, $f$, $\gamma$, and all dimensionless quantities.
2. **Bare prime** ($X'$) — the quantity is an Arc I (Tier 1) corridor operator, representation, or structural projection. Examples: $M' = 1/\lambda$, $q' = fX$, $\sqrt{G'}$. Authority: the Arc I Canonical Glossary and Papers I–V.
3. **SI-left-subscripted prime** (${}_{\mathrm{SI}}X'$) — the quantity is a Tier 3 $\ell_m$-reduced dimensional form. Examples: ${}_{\mathrm{SI}}M' = m/\ell_m$, ${}_{\mathrm{SI}}E' = E/\ell_m$, ${}_{\mathrm{SI}}G' = G\,\ell_m$. Authority: this volume.

Throughout this volume, quantities obtained by $\ell_m$-reduction carry the left subscript SI before the prime (${}_{\mathrm{SI}}M'$, ${}_{\mathrm{SI}}E'$, ${}_{\mathrm{SI}}P'$, etc.). Quantities whose dimensions are unchanged from SI retain standard notation without prime. The bare-primed form does not appear in this volume; readers who encounter a bare prime elsewhere in the LMR program should consult the Arc I Canonical Glossary, not this document.

The method is operational:

1. Inspect each term of an SI equation.
2. Identify occurrences of kilogram dependence.
3. Eliminate kg by multiplication or division by $\ell_m$.
4. Admit the reduced quantities (denoted by ${}_{\mathrm{SI}}X'$) into the equation.

No geometric interpretation is assumed. No structural ontology is invoked. All results follow from dimensional analysis alone.

### Disclaimer (Scope and Discipline)

**Tier discipline.** This volume belongs to the Correlation Layer only. It performs $\ell_m$-reduction on SI expressions and records the resulting kg-free forms.

**No ontology import.** No statement in this volume asserts structural primitives, corridor operators, engagement factors, admissibility rules, or any geometric realization claim. Those belong exclusively to the Structural Grammar (Papers I–V).

**No dynamics import.** No statement in this volume asserts evolution laws, stochastic hazards, transport equations, or probabilistic interpretation. Those belong exclusively to the Overlay Layer (Companion Volume I), where any overlay assumptions must be declared explicitly.

**What reduction can and cannot do.** $\ell_m$-reduction can remove kilogram dependence and reorganize dimensional faces. It cannot, by itself, derive new physical content. Any interpretive conclusion must arise from Tier 1 (Structural Grammar) or Tier 2 (Overlay Layer), not from the reduction procedure alone.

**Notation.** The SI-left-subscripted prime indicates "$\ell_m$-reduced dimensional face." It does not indicate a new entity, a new regime, or a new measurement. It is a bookkeeping marker for unit elimination.

# 1 $\ell_m$-Reduction Discipline

**Orientation.** This volume performs an algebraic elimination of kilogram dependence from SI equations. No new physical assumptions are introduced. The procedure is a dimensional translation based on the bridge quantity $\ell_m := h/c$. All results are SI-equivalent statements rewritten in kg-free form.

## 1.1 Bridge Quantity and Notation

**Definition 1.1** (Bridge Quantity)**.** Define the bridge quantity

$$\ell_m := \frac{h}{c},$$

with SI units kg·m.

**Definition 1.2** (Prime Convention)**.** A left-subscripted SI prime (${}_{\mathrm{SI}}X'$) denotes a quantity expressed in $\ell_m$-reduced (kg-free) units. Quantities whose dimensions are unchanged under reduction retain standard notation. The bare-primed form $X'$ is reserved for Arc I (Tier 1) corridor and representation quantities and does not appear in this volume.

## 1.2 Core Replacement Rules

**Model Rule 1.1** (R1: Replace $h$ by $\ell_m c$)**.** Whenever $h$ appears in an SI expression, substitute

$$h = \ell_m c.$$

**Model Rule 1.2** (R2: Replace $m$ by ${}_{\mathrm{SI}}M'\,\ell_m$)**.** Whenever a mass $m$ appears, define its $\ell_m$-reduced form by

$${}_{\mathrm{SI}}M' := \frac{m}{\ell_m} \iff m = {}_{\mathrm{SI}}M'\,\ell_m.$$

**Model Rule 1.3** (R3: Reduce energies by dividing by $\ell_m$)**.** Whenever an energy $E$ appears, define the reduced energy

$${}_{\mathrm{SI}}E' := \frac{E}{\ell_m}.$$

**Model Rule 1.4** (R4: Reduce momenta by dividing by $\ell_m$)**.** Whenever a momentum $p$ appears, define the reduced momentum

$${}_{\mathrm{SI}}P' := \frac{p}{\ell_m}.$$

## 1.3 Cancellation Rule (No Residual $\ell_m$)

**Proposition 1.1** (Cancellation Principle)**.** After applying Rules R1–R4 in the reductions treated in this volume, final kg-free forms should be written with all explicit $\ell_m$ factors cancelled whenever the corresponding reduced quantities have been defined.

*Remark* 1.1 (Where $\ell_m$ is allowed to appear)*.* The bridge quantity $\ell_m$ is permitted only as an intermediate eliminator in the reduction process. Final forms should be written without explicit $\ell_m$ whenever cancellation is possible.

### 1.4 Pairing Discipline: The Compton Identity

**Definition 1.3** (Compton Pairing Identity)**.** The SI Compton wavelength is defined by

$$\lambda_C := \frac{h}{mc}.$$

Applying Rule R1 gives

$$\lambda_C = \frac{\ell_m}{m}.$$

Applying Rule R2 gives

$$\lambda_C = \frac{1}{{}_{\mathrm{SI}}M'}.$$

Hence the pairing identity

$${}_{\mathrm{SI}}M'\,\lambda_C = 1$$

holds only for the Compton pairing: $\lambda_C$ is the reciprocal face of the $\ell_m$-reduced mass ${}_{\mathrm{SI}}M'$.

*Remark* 1.2 (Do not generalize the pairing)*.* The identity ${}_{\mathrm{SI}}M'\,\lambda = 1$ is not asserted for arbitrary lengths $\lambda$. It applies specifically and only to the Compton wavelength $\lambda_C$ defined from the same SI mass $m$.

### 1.5 Operational Procedure

1. Write the SI equation.
2. Substitute $h = \ell_m c$ (R1).
3. Substitute $m = {}_{\mathrm{SI}}M'\,\ell_m$ wherever $m$ appears (R2).
4. If the equation contains $E$ or $p$, introduce ${}_{\mathrm{SI}}E' = E/\ell_m$ (R3) and ${}_{\mathrm{SI}}P' = p/\ell_m$ (R4).
5. Cancel all explicit $\ell_m$ factors.
6. Present the final reduced equation in terms of primed quantities.

**Hierarchy Reminder (Chapter 1 — $\ell_m$-Reduction Discipline)**

**Bridge**
- $\ell_m := h/c$ (units kg·m).

**Core Rules**
- R1: $h = \ell_m c$.
- R2: ${}_{\mathrm{SI}}M' := m/\ell_m$.
- R3: ${}_{\mathrm{SI}}E' := E/\ell_m$.
- R4: ${}_{\mathrm{SI}}P' := p/\ell_m$.

**Validity**
- All explicit $\ell_m$ factors must cancel in final reduced forms.

**Pairing Discipline**
- ${}_{\mathrm{SI}}M'\,\lambda_C = 1$ holds only for the Compton wavelength $\lambda_C = h/(mc)$.
- Do not generalize ${}_{\mathrm{SI}}M'\,\lambda = 1$ to arbitrary lengths.

## 2 Dimensional Inventory Under $\ell_m$-Reduction

**Purpose.** Before applying $\ell_m$-reduction to specific physical theories, it is necessary to identify which SI quantities contain kilogram dependence and which do not. This chapter establishes that inventory.

### 2.1 Quantities Unaffected by $\ell_m$-Reduction

**Proposition 2.1.** Any quantity whose SI dimension does not contain kg is unchanged under $\ell_m$-reduction.

Examples include:

$$c, \quad v, \quad \lambda, \quad t, \quad f, \quad \gamma = \frac{1}{\sqrt{1 - v^2/c^2}}.$$

These quantities retain their notation and dimensional form.

*Remark* 2.1. Dimensionless quantities are invariant under $\ell_m$-reduction.

### 2.2 Mass

SI mass carries one unit of kilogram:

$$[m] = \text{kg}.$$

Applying Rule R2:

$$_{\text{SI}}M' := \frac{m}{\ell_m}, \qquad m = {}_{\text{SI}}M' \, \ell_m.$$

Since $[\ell_m] = \text{kg} \cdot \text{m}$,

$$[_{\text{SI}}M'] = \frac{\text{kg}}{\text{kg} \cdot \text{m}} = \text{m}^{-1}.$$

Thus $\ell_m$-reduced mass carries inverse length dimension.

### 2.3 Energy

SI energy:

$$[E] = \text{kg} \cdot \text{m}^2/\text{s}^2.$$

Apply Rule R3:

$$_{\text{SI}}E' := \frac{E}{\ell_m}.$$

Then

$$[_{\text{SI}}E'] = \frac{\text{kg} \cdot \text{m}^2/\text{s}^2}{\text{kg} \cdot \text{m}} = \text{m}/\text{s}^2.$$

Energy reduces to a length–time quantity with no mass dependence.

### 2.4 Momentum

SI momentum:

$$[p] = \text{kg} \cdot \text{m}/\text{s}.$$

Apply Rule R4:

$$_{\text{SI}}P' := \frac{p}{\ell_m}.$$

Then

$$[_{\mathrm{SI}}P'] = \frac{\mathrm{kg} \cdot \mathrm{m/s}}{\mathrm{kg} \cdot \mathrm{m}} = \mathrm{s}^{-1}.$$

Reduced momentum carries inverse time dimension.

### 2.5 Force

SI force:

$$[F] = \mathrm{kg} \cdot \mathrm{m/s}^2.$$

Define reduced force:

$$_{\mathrm{SI}}F' := \frac{F}{\ell_m}.$$

Then

$$[_{\mathrm{SI}}F'] = \frac{\mathrm{kg} \cdot \mathrm{m/s}^2}{\mathrm{kg} \cdot \mathrm{m}} = \mathrm{s}^{-2}.$$

Force reduces to inverse time squared.

### 2.6 Planck Constant

By Rule R1:

$$h = \ell_m c.$$

Planck's constant therefore carries no independent role after $\ell_m$-reduction; it is absorbed into the bridge quantity.

### 2.7 Summary of Dimensional Shifts

| **Quantity** | **SI Dimension** | **$\ell_m$-Reduced Dimension** |
|---|---|---|
| Mass $m$ | kg | $\mathrm{m}^{-1}$ |
| Energy $E$ | $\mathrm{kg{\cdot}m^2/s^2}$ | $\mathrm{m/s^2}$ |
| Momentum $p$ | kg·m/s | $\mathrm{s}^{-1}$ |
| Force $F$ | $\mathrm{kg{\cdot}m/s^2}$ | $\mathrm{s}^{-2}$ |

All kilogram dependence has been eliminated.

**Hierarchy Reminder (Chapter 2 — Dimensional Inventory)**

**Unchanged**

- $c$, $v$, $\lambda$, $t$, $f$, $\gamma$.
- All dimensionless quantities.

**Reduced**

- $_{\mathrm{SI}}M' = m/\ell_m$ ($\mathrm{m}^{-1}$).
- $_{\mathrm{SI}}E' = E/\ell_m$ ($\mathrm{m/s^2}$).
- $_{\mathrm{SI}}P' = p/\ell_m$ ($\mathrm{s}^{-1}$).
- $_{\mathrm{SI}}F' = F/\ell_m$ ($\mathrm{s}^{-2}$).

**Bridge**

- $h = \ell_m c$.
- All explicit $\ell_m$ must cancel in final reduced equations.

## 3 Mechanics Under $\ell_m$-Reduction

**Purpose.** This chapter applies the $\ell_m$-reduction discipline to elementary mechanical relations. No new assumptions are introduced. All results follow directly from the replacement rules established in Chapter 1.

### 3.1 Linear Momentum

In SI,

$$p = mv.$$

Apply Rule R2:

$$m = {}_{\mathrm{SI}}M' \, \ell_m.$$

Then

$$p = {}_{\mathrm{SI}}M' \, \ell_m \, v.$$

Define reduced momentum (Rule R4):

$${}_{\mathrm{SI}}P' := \frac{p}{\ell_m}.$$

Divide by $\ell_m$:

$${}_{\mathrm{SI}}P' = {}_{\mathrm{SI}}M' \, v.$$

This is the $\ell_m$-reduced momentum relation.

### 3.2 Rest Energy

In SI,

$$E = mc^2.$$

Apply Rule R2:

$$E = {}_{\mathrm{SI}}M' \, \ell_m c^2.$$

Define reduced energy (Rule R3): ${}_{\mathrm{SI}}E' := E/\ell_m$. Divide by $\ell_m$:

$${}_{\mathrm{SI}}E' = {}_{\mathrm{SI}}M' \, c^2.$$

No explicit $\ell_m$ remains.

### 3.3 Kinetic Energy

In SI,

$$E_k = \tfrac{1}{2}mv^2.$$

Substitute $m = {}_{\mathrm{SI}}M' \, \ell_m$:

$$E_k = \tfrac{1}{2} \, {}_{\mathrm{SI}}M' \, \ell_m \, v^2.$$

Define reduced kinetic energy:

$${}_{\mathrm{SI}}E'_k := \frac{E_k}{\ell_m}.$$

Divide:

$${}_{\mathrm{SI}}E'_k = \tfrac{1}{2} \, {}_{\mathrm{SI}}M' \, v^2.$$

## 3.4 Relativistic Energy

In SI,

$$E = \gamma mc^2.$$

Substitute:

$$E = \gamma \, {}_{\mathrm{SI}}M' \, \ell_m c^2.$$

Define reduced energy: ${}_{\mathrm{SI}}E' := E/\ell_m$. Then

$${}_{\mathrm{SI}}E' = \gamma \, {}_{\mathrm{SI}}M' \, c^2.$$

The Lorentz factor $\gamma$ remains unchanged.

## 3.5 Energy–Momentum Relation

In SI,

$$E^2 = p^2c^2 + m^2c^4.$$

Substitute

$$E = {}_{\mathrm{SI}}E'\ell_m, \quad p = {}_{\mathrm{SI}}P'\ell_m, \quad m = {}_{\mathrm{SI}}M'\ell_m.$$

Then

$$({}_{\mathrm{SI}}E'\ell_m)^2 = ({}_{\mathrm{SI}}P'\ell_m)^2c^2 + ({}_{\mathrm{SI}}M'\ell_m)^2c^4.$$

Factor $\ell_m^2$:

$$\ell_m^2 {}_{\mathrm{SI}}E'^2 = \ell_m^2 {}_{\mathrm{SI}}P'^2c^2 + \ell_m^2 {}_{\mathrm{SI}}M'^2c^4.$$

Cancel $\ell_m^2$:

$${}_{\mathrm{SI}}E'^2 = {}_{\mathrm{SI}}P'^2c^2 + {}_{\mathrm{SI}}M'^2c^4.$$

The algebraic form is preserved exactly.

## 3.6 Dimensional Interpretation

From Chapter 2:

$$[{}_{\mathrm{SI}}M'] = \mathrm{m}^{-1}, \quad [{}_{\mathrm{SI}}P'] = \mathrm{s}^{-1}, \quad [{}_{\mathrm{SI}}E'] = \mathrm{m/s}^2.$$

Thus ${}_{\mathrm{SI}}P' = {}_{\mathrm{SI}}M' \, v$ and ${}_{\mathrm{SI}}E' = {}_{\mathrm{SI}}M' \, c^2$ are dimensionally consistent. No kilogram remains in any mechanical relation.

**Hierarchy Reminder (Chapter 3 — Mechanics)**

**Reduced Relations**

- ${}_{\mathrm{SI}}P' = {}_{\mathrm{SI}}M' \, v$.
- ${}_{\mathrm{SI}}E' = {}_{\mathrm{SI}}M' \, c^2$.
- ${}_{\mathrm{SI}}E'_k = \frac{1}{2} \, {}_{\mathrm{SI}}M' \, v^2$.
- ${}_{\mathrm{SI}}E'^2 = {}_{\mathrm{SI}}P'^2c^2 + {}_{\mathrm{SI}}M'^2c^4$.

**Properties**

- Algebraic structure preserved.
- $\gamma$ unchanged (dimensionless).
- All explicit $\ell_m$ cancelled.
- No geometric interpretation invoked.

# 4 Wave Relations Under $\ell_m$-Reduction

**Purpose.** This chapter applies $\ell_m$-reduction to the wave relations of standard quantum mechanics. No structural assumptions are introduced. All results follow from dimensional replacement rules.

## 4.1 de Broglie Relation

In SI,

$$\lambda_{\mathrm{dB}} = \frac{h}{p}.$$

Apply Rule R1:

$$\lambda_{\mathrm{dB}} = \frac{\ell_m c}{p}.$$

Apply Rule R4: $p = {}_{\mathrm{SI}}P'\,\ell_m$. Substitute:

$$\lambda_{\mathrm{dB}} = \frac{\ell_m c}{{}_{\mathrm{SI}}P'\,\ell_m}.$$

Cancel $\ell_m$:

$$\lambda_{\mathrm{dB}} = \frac{c}{{}_{\mathrm{SI}}P'}.$$

Thus the reduced momentum relation becomes

$${}_{\mathrm{SI}}P' = \frac{c}{\lambda_{\mathrm{dB}}}.$$

No kilogram remains.

## 4.2 Planck Relation

In SI,

$$E = hf.$$

Apply Rule R1:

$$E = \ell_m c f.$$

Apply Rule R3: ${}_{\mathrm{SI}}E' = E/\ell_m$. Cancel:

$${}_{\mathrm{SI}}E' = cf.$$

Thus

$$f = \frac{{}_{\mathrm{SI}}E'}{c}.$$

## 4.3 Consistency with Mechanics

From Chapter 3, ${}_{\mathrm{SI}}E' = {}_{\mathrm{SI}}M'\,c^2$. Equating with the Planck form:

$${}_{\mathrm{SI}}M'\,c^2 = cf.$$

Divide by $c$:

$${}_{\mathrm{SI}}M'\,c = f.$$

If a characteristic wavelength $\lambda$ is defined by $f = c/\lambda$, then

$${}_{\mathrm{SI}}M' = \frac{1}{\lambda}.$$

This relation must be interpreted with care; see the pairing discipline below.

### 4.4 Compton Wavelength

In SI,

$$\lambda_C = \frac{h}{mc}.$$

Apply Rule R1: $\lambda_C = \ell_m c/(mc) = \ell_m/m$. Apply Rule R2:

$$\lambda_C = \frac{1}{{}_{\mathrm{SI}}M'}.$$

Thus the pairing identity

$${}_{\mathrm{SI}}M'\,\lambda_C = 1$$

holds specifically for the Compton wavelength.

### 4.5 Pairing Discipline

The identity ${}_{\mathrm{SI}}M' = 1/\lambda$ does not hold for arbitrary length scales. It holds only for the Compton wavelength defined from the same SI mass parameter.

The de Broglie wavelength satisfies instead

$$\lambda_{\mathrm{dB}} = \frac{c}{{}_{\mathrm{SI}}P'},$$

which depends on velocity through ${}_{\mathrm{SI}}P' = {}_{\mathrm{SI}}M'\,v$. Thus

$$\lambda_{\mathrm{dB}} = \frac{c}{{}_{\mathrm{SI}}M'\,v}.$$

Formally, $\lambda_{\mathrm{dB}}$ approaches the Compton pairing only when the momentum scale satisfies ${}_{\mathrm{SI}}P' = {}_{\mathrm{SI}}M'c$. No physical massive-particle identification is implied.

### 4.6 Summary

All wave relations reduce without residual mass units. Planck's constant is absorbed into $\ell_m$. Momentum and mass relations remain structurally consistent.

**Hierarchy Reminder (Chapter 4 — Wave Relations)**

**Reduced Forms**
- ${}_{\mathrm{SI}}P' = c/\lambda_{\mathrm{dB}}$.
- ${}_{\mathrm{SI}}E' = cf$.
- $\lambda_C = 1/{}_{\mathrm{SI}}M'$.

**Pairing Discipline**
- ${}_{\mathrm{SI}}M'\,\lambda_C = 1$ applies only to the Compton wavelength.
- $\lambda_{\mathrm{dB}}$ depends on velocity via ${}_{\mathrm{SI}}P' = {}_{\mathrm{SI}}M'\,v$.
- No arbitrary length may be paired with ${}_{\mathrm{SI}}M'$.

**Properties**
- No explicit $\ell_m$ remains.
- Algebraic structure preserved.
- No geometric interpretation invoked.

# 5 Electromagnetism Under $\ell_m$-Reduction

**Purpose.** This chapter eliminates kilogram dependence from electromagnetic constants and equations. No reinterpretation is introduced. All reductions follow from the rules established in Chapter 1.

## 5.1 Vacuum Permittivity

In SI,

$$[\varepsilon_0] = \frac{\mathrm{C}^2}{\mathrm{N}\cdot\mathrm{m}^2} = \frac{\mathrm{C}^2\,\mathrm{s}^2}{\mathrm{kg}\cdot\mathrm{m}^3}.$$

The dimension contains one inverse mass factor. Define the reduced permittivity:

$${}_{\mathrm{SI}}\varepsilon_0' := \varepsilon_0\,\ell_m.$$

Then

$$[{}_{\mathrm{SI}}\varepsilon_0'] = \frac{\mathrm{C}^2\,\mathrm{s}^2}{\mathrm{m}^2}.$$

All kilogram dependence is eliminated.

## 5.2 Vacuum Permeability

In SI,

$$[\mu_0] = \frac{\mathrm{N}}{\mathrm{A}^2} = \frac{\mathrm{kg}\cdot\mathrm{m}}{\mathrm{C}^2}.$$

The dimension contains one direct mass factor. Define the reduced permeability:

$${}_{\mathrm{SI}}\mu_0' := \frac{\mu_0}{\ell_m}.$$

Then

$$[{}_{\mathrm{SI}}\mu_0'] = \frac{1}{\mathrm{C}^2}.$$

No kilogram remains.

## 5.3 Maxwell Closure

In SI,

$$c^2 = \frac{1}{\mu_0\varepsilon_0}.$$

Substitute $\mu_0 = {}_{\mathrm{SI}}\mu_0'\,\ell_m$ and $\varepsilon_0 = {}_{\mathrm{SI}}\varepsilon_0'/\ell_m$. Then

$$c^2 = \frac{1}{({}_{\mathrm{SI}}\mu_0'\,\ell_m)\,({}_{\mathrm{SI}}\varepsilon_0'/\ell_m)}.$$

Cancel $\ell_m$:

$$c^2 = \frac{1}{{}_{\mathrm{SI}}\mu_0'\,{}_{\mathrm{SI}}\varepsilon_0'}.$$

The algebraic form is preserved.

### 5.4 Coulomb's Law

In SI,

$$F = \frac{1}{4\pi\varepsilon_0}\,\frac{q_1 q_2}{r^2}.$$

Define reduced force ${}_{\mathrm{SI}}F' := F/\ell_m$ and ${}_{\mathrm{SI}}\varepsilon_0' := \varepsilon_0\,\ell_m$. Then

$${}_{\mathrm{SI}}F' = \frac{1}{4\pi\,{}_{\mathrm{SI}}\varepsilon_0'}\,\frac{q_1 q_2}{r^2}.$$

All explicit mass factors cancel.

### 5.5 Charge

Electric charge does not contain explicit mass in its SI base dimension. No reduction is applied directly to $q$. Mass dependence in electromagnetic expressions enters only through $\varepsilon_0$ and $\mu_0$.

### 5.6 Fine-Structure Constant

In SI,

$$\alpha = \frac{e^2}{4\pi\varepsilon_0\,\hbar c}.$$

Rewrite $\hbar = h/(2\pi)$ and substitute $h = \ell_m c$:

$$\alpha = \frac{e^2}{2\,\varepsilon_0\,\ell_m\,c^2}.$$

Substitute $\varepsilon_0 = {}_{\mathrm{SI}}\varepsilon_0'/\ell_m$:

$$\alpha = \frac{e^2}{2\,({}_{\mathrm{SI}}\varepsilon_0'/\ell_m)\,\ell_m\,c^2}.$$

Cancel $\ell_m$:

$$\alpha = \frac{e^2}{2\,{}_{\mathrm{SI}}\varepsilon_0'\,c^2}.$$

The fine-structure constant is therefore expressible without explicit mass dependence.

### 5.7 Normalized Charge Scale

Define a normalized charge scale by

$${}_{\mathrm{SI}}q'^{\,2} := {}_{\mathrm{SI}}\varepsilon_0'\,c^2.$$

Then

$$\alpha = \frac{1}{2}\left(\frac{e}{{}_{\mathrm{SI}}q'}\right)^2.$$

This expression is algebraically equivalent to the SI definition. No numerical prediction is introduced.

**Hierarchy Reminder (Chapter 5 — Electromagnetism)**

**Reduced Constants**

- ${}_{\mathrm{SI}}\varepsilon_0' = \varepsilon_0\,\ell_m$.

- ${}_{\mathrm{SI}}\mu_0' = \mu_0/\ell_m$.

**Closure**

- $c^2 = 1/({}_{\mathrm{SI}}\mu_0'\,{}_{\mathrm{SI}}\varepsilon_0')$.

**Coulomb Law**

- ${}_{\mathrm{SI}}F' = \frac{1}{4\pi\,{}_{\mathrm{SI}}\varepsilon_0'}\,\frac{q_1 q_2}{r^2}$.

**Fine-Structure Constant**

- $\alpha = e^2/(2\,{}_{\mathrm{SI}}\varepsilon_0'\,c^2)$.
- $\alpha = \frac{1}{2}(e/{}_{\mathrm{SI}}q')^2$ with ${}_{\mathrm{SI}}q'^{\,2} = {}_{\mathrm{SI}}\varepsilon_0'\,c^2$.

**Properties**

- No explicit kilogram remains.
- Algebraic structure preserved.
- No geometric interpretation invoked.

## 6 Atomic Scales Under $\ell_m$-Reduction

**Purpose.** This chapter applies $\ell_m$-reduction to standard atomic length and frequency scales. All results are algebraic translations of their SI definitions. No structural interpretation is introduced.

### 6.1 Classical Electron Radius

In SI,

$$r_e = \frac{e^2}{4\pi\varepsilon_0\, mc^2}.$$

Substitute $m = {}_{\mathrm{SI}}M'\,\ell_m$ and $\varepsilon_0 = {}_{\mathrm{SI}}\varepsilon_0'/\ell_m$:

$$r_e = \frac{e^2}{4\pi\,({}_{\mathrm{SI}}\varepsilon_0'/\ell_m)\,({}_{\mathrm{SI}}M'\,\ell_m)\,c^2}.$$

Cancel $\ell_m$:

$$r_e = \frac{e^2}{4\pi\,{}_{\mathrm{SI}}\varepsilon_0'\,{}_{\mathrm{SI}}M'\,c^2}.$$

Using $\alpha = e^2/(2\,{}_{\mathrm{SI}}\varepsilon_0'\,c^2)$, we obtain

$$r_e = \frac{\alpha}{2\pi\,{}_{\mathrm{SI}}M'}.$$

No mass unit remains.

### 6.2 Bohr Radius

In SI,

$$a_0 = \frac{4\pi\varepsilon_0\,\hbar^2}{m\,e^2}.$$

Rewrite $\hbar = h/(2\pi)$ and substitute $h = \ell_m c$:

$$\hbar = \frac{\ell_m c}{2\pi}.$$

Then

$$a_0 = \frac{4\pi\,({}_{\mathrm{SI}}\varepsilon_0'/\ell_m)\,(\ell_m^2 c^2/4\pi^2)}{({}_{\mathrm{SI}}M'\,\ell_m)\,e^2}.$$

Cancel $\ell_m$ factors:

$$a_0 = \frac{{}_{\mathrm{SI}}\varepsilon_0'\,c^2}{\pi\,{}_{\mathrm{SI}}M'\,e^2}.$$

Using $\alpha = e^2/(2\,{}_{\mathrm{SI}}\varepsilon_0'\,c^2)$, we obtain

$$a_0 = \frac{1}{2\pi\,\alpha\,{}_{\mathrm{SI}}M'}.$$

All kilogram dependence has cancelled.

### 6.3 Rydberg Constant

In SI,

$$R_\infty = \frac{me^4}{8\,\varepsilon_0^2\,h^3 c}.$$

Substitute $m = {}_{\mathrm{SI}}M'\,\ell_m$, $h = \ell_m c$, and $\varepsilon_0 = {}_{\mathrm{SI}}\varepsilon_0'/\ell_m$. After cancellation of $\ell_m$ factors, the result reduces to

$$R_\infty = \tfrac{1}{2}\,\alpha^2\,{}_{\mathrm{SI}}M'.$$

No kilogram remains.

### 6.4 Dimensional Pattern

The reduced expressions exhibit a consistent structure:

$$r_e \propto \frac{\alpha}{{}_{\mathrm{SI}}M'}, \qquad a_0 \propto \frac{1}{\alpha\,{}_{\mathrm{SI}}M'}, \qquad R_\infty \propto \alpha^2\,{}_{\mathrm{SI}}M'.$$

Length scales are inversely proportional to ${}_{\mathrm{SI}}M'$. Frequency scales are proportional to ${}_{\mathrm{SI}}M'$. No additional assumptions are required.

**Hierarchy Reminder (Chapter 6 — Atomic Scales)**

**Reduced Lengths**
- $r_e = \alpha/(2\pi\,{}_{\mathrm{SI}}M')$.
- $a_0 = 1/(2\pi\,\alpha\,{}_{\mathrm{SI}}M')$.

**Reduced Frequency Scale**
- $R_\infty = \frac{1}{2}\,\alpha^2\,{}_{\mathrm{SI}}M'$.

**Properties**
- All $\ell_m$ factors cancel.
- No kilogram remains.
- Algebraic structure preserved.
- No geometric interpretation invoked.

# 7 Schrödinger Equation Under $\ell_m$-Reduction

**Purpose.** This chapter applies $\ell_m$-reduction to the non-relativistic Schrödinger equation. The algebraic structure of the equation is preserved. No physical reinterpretation is introduced.

## 7.1 Time-Dependent Schrödinger Equation

In SI,

$$i\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\psi + V\psi.$$

Rewrite $\hbar = h/(2\pi)$ and substitute $h = \ell_m c$:

$$\hbar = \frac{\ell_m c}{2\pi}.$$

## 7.2 Left-Hand Side

$$i\hbar\frac{\partial\psi}{\partial t} = i\,\frac{\ell_m c}{2\pi}\,\frac{\partial\psi}{\partial t}.$$

Define reduced potential

$${}_{\mathrm{SI}}V' := \frac{V}{\ell_m}.$$

Define reduced energy scale implicitly via ${}_{\mathrm{SI}}E' = E/\ell_m$. Divide the entire equation by $\ell_m$. The left-hand side becomes

$$i\,\frac{c}{2\pi}\,\frac{\partial\psi}{\partial t}.$$

## 7.3 Kinetic Term

In SI,

$$-\frac{\hbar^2}{2m} = -\frac{1}{2m}\left(\frac{\ell_m c}{2\pi}\right)^2.$$

Substitute $m = {}_{\mathrm{SI}}M'\,\ell_m$:

$$-\frac{1}{2\,{}_{\mathrm{SI}}M'\,\ell_m}\,\frac{\ell_m^2\,c^2}{4\pi^2}.$$

Cancel one $\ell_m$:

$$-\frac{\ell_m\,c^2}{8\pi^2\,{}_{\mathrm{SI}}M'}.$$

Divide by $\ell_m$:

$$-\frac{c^2}{8\pi^2\,{}_{\mathrm{SI}}M'}.$$

Thus the kinetic operator becomes

$$-\frac{c^2}{8\pi^2\,{}_{\mathrm{SI}}M'}\,\nabla^2\psi.$$

### 7.4 Reduced Equation

The $\ell_m$-reduced time-dependent Schrödinger equation is

$$i\,\frac{c}{2\pi}\,\frac{\partial\psi}{\partial t} = -\frac{c^2}{8\pi^2\,{}_{\mathrm{SI}}M'}\,\nabla^2\psi + {}_{\mathrm{SI}}V'\psi.$$

No explicit $\ell_m$ remains.

### 7.5 Time-Independent Form

In SI,

$$-\frac{\hbar^2}{2m}\nabla^2\psi + V\psi = E\psi.$$

After $\ell_m$-reduction:

$$-\frac{c^2}{8\pi^2\,{}_{\mathrm{SI}}M'}\,\nabla^2\psi + {}_{\mathrm{SI}}V'\psi = {}_{\mathrm{SI}}E'\psi.$$

The algebraic structure is unchanged.

### 7.6 Observations

- The equation retains its differential structure.
- All kilogram dependence cancels.
- $\hbar$ is absorbed into $\ell_m$ and eliminated.
- The only mass parameter remaining is ${}_{\mathrm{SI}}M'$.

**Hierarchy Reminder (Chapter 7 — Schrödinger)**

**Reduced Equation**

$$i\,\frac{c}{2\pi}\,\frac{\partial\psi}{\partial t} = -\frac{c^2}{8\pi^2\,{}_{\mathrm{SI}}M'}\,\nabla^2\psi + {}_{\mathrm{SI}}V'\psi.$$

**Properties**

- All $\ell_m$ factors cancel.
- $\hbar$ does not appear.
- Only ${}_{\mathrm{SI}}M'$ carries mass information.
- Algebraic structure preserved.
- No geometric interpretation invoked.

# 8 Klein–Gordon Equation Under $\ell_m$-Reduction

**Purpose.** This chapter applies $\ell_m$-reduction to the relativistic Klein–Gordon equation. The algebraic structure of the equation is preserved. No reinterpretation is introduced.

## 8.1 SI Form

In SI units, the Klein–Gordon equation reads

$$\left(\frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \nabla^2 + \frac{m^2 c^2}{\hbar^2}\right)\psi = 0.$$

## 8.2 Substitution Rules

Apply $\ell_m$-reduction substitutions:

$$m = {}_{\mathrm{SI}}M'\,\ell_m, \qquad \hbar = \frac{\ell_m c}{2\pi}.$$

## 8.3 Mass Term Reduction

The mass term becomes

$$\frac{m^2 c^2}{\hbar^2} = \frac{({}_{\mathrm{SI}}M'\,\ell_m)^2\,c^2}{(\ell_m c/2\pi)^2}.$$

Expand numerator and denominator:

$$= \frac{{}_{\mathrm{SI}}M'^{\,2}\,\ell_m^2\,c^2}{\ell_m^2\,c^2/(4\pi^2)}.$$

Cancel $\ell_m^2$ and $c^2$:

$$= 4\pi^2\,{}_{\mathrm{SI}}M'^{\,2}.$$

Thus the mass term reduces to $4\pi^2\,{}_{\mathrm{SI}}M'^{\,2}$. No kilogram remains.

## 8.4 Reduced Klein–Gordon Equation

The $\ell_m$-reduced equation becomes

$$\left(\frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \nabla^2 + 4\pi^2\,{}_{\mathrm{SI}}M'^{\,2}\right)\psi = 0.$$

## 8.5 Observations

- The differential operator is unchanged.
- All $\ell_m$ factors cancel exactly.
- $\hbar$ disappears entirely.
- The mass term becomes purely quadratic in ${}_{\mathrm{SI}}M'$.
- The equation retains identical relativistic structure.

**Hierarchy Reminder (Chapter 8 — Klein–Gordon)**

**Reduced Equation**

$$\left(\frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \nabla^2 + 4\pi^2\,{}_{\mathrm{SI}}M'^2\right)\psi = 0.$$

**Properties**

- $\hbar$ eliminated.
- No kilogram remains.
- Mass term reduces to $4\pi^2\,{}_{\mathrm{SI}}M'^2$.
- Algebraic structure preserved.

# 9 Dirac Equation Under $\ell_m$-Reduction

**Purpose.** This chapter applies $\ell_m$-reduction to the relativistic Dirac equation. The algebraic structure is preserved. No new physical assumptions are introduced.

## 9.1 SI Form

One standard covariant form of the free Dirac equation is

$$\big(i\hbar c\,\gamma^\mu\partial_\mu - mc^2\big)\psi = 0,$$

where $\gamma^\mu$ are the Dirac matrices and $\partial_\mu$ is the four-gradient. Equivalently, dividing by $\hbar c$:

$$\Big(i\gamma^\mu\partial_\mu - \frac{mc}{\hbar}\Big)\psi = 0.$$

## 9.2 Substitution Rules

Apply $\ell_m$-reduction substitutions:

$$m = {}_{\mathrm{SI}}M'\,\ell_m, \qquad \hbar = \frac{\ell_m c}{2\pi}.$$

## 9.3 Mass Term Reduction

Reduce the SI mass frequency factor:

$$\frac{mc}{\hbar} = \frac{({}_{\mathrm{SI}}M'\,\ell_m)\,c}{\ell_m c/(2\pi)}.$$

Cancel $\ell_m c$:

$$\frac{mc}{\hbar} = 2\pi\,{}_{\mathrm{SI}}M'.$$

No kilogram remains.

## 9.4 Reduced Dirac Equation

Substituting into the divided form gives the $\ell_m$-reduced Dirac equation:

$$\big(i\gamma^\mu\partial_\mu - 2\pi\,{}_{\mathrm{SI}}M'\big)\psi = 0.$$

This is the cleanest reduced statement: it preserves the first-order relativistic structure while eliminating all kilogram dependence.

## 9.5 Alternative (Undivided) Reduced Form

Starting from the undivided SI form

$$\big(i\hbar c\,\gamma^\mu\partial_\mu - mc^2\big)\psi = 0,$$

substitute $m = {}_{\mathrm{SI}}M'\,\ell_m$ and $\hbar = \ell_m c/(2\pi)$:

$$\Big(i\,\frac{\ell_m c}{2\pi}\,c\,\gamma^\mu\partial_\mu - {}_{\mathrm{SI}}M'\,\ell_m\,c^2\Big)\psi = 0.$$

Factor $\ell_m c^2$:

$$\ell_m c^2\Big(\frac{i}{2\pi}\gamma^\mu\partial_\mu - {}_{\mathrm{SI}}M'\Big)\psi = 0.$$

Since $\ell_m c^2 \neq 0$, this is equivalent to

$$\big(i\gamma^\mu\partial_\mu - 2\pi\,{}_{\mathrm{SI}}M'\big)\psi = 0,$$

which matches the reduced equation above.

### 9.6 Observations

- The Dirac matrices $\gamma^\mu$ are dimensionless and unchanged.
- All $\ell_m$ dependence cancels exactly.
- The SI factor $mc/\hbar$ reduces to $2\pi\,{}_{\mathrm{SI}}M'$.
- The equation remains first-order in spacetime derivatives.

### 9.7 Schrödinger as the Low-Velocity Limit of the Reduced Invariant

**Starting Point.** From Chapter 3 (Energy–Momentum Relation), the $\ell_m$-reduced relativistic invariant is

$${}_{\mathrm{SI}}E'^{\,2} = {}_{\mathrm{SI}}P'^{\,2}\,c^2 + {}_{\mathrm{SI}}M'^{\,2}\,c^4.$$

This relation contains no kilogram dependence and serves as the relativistic backbone of the reduced formulation.

#### Low-Momentum Expansion

For ${}_{\mathrm{SI}}P'^{\,2} \ll {}_{\mathrm{SI}}M'^{\,2}\,c^2$, expand the square root:

$${}_{\mathrm{SI}}E' = \sqrt{{}_{\mathrm{SI}}M'^{\,2}c^4 + {}_{\mathrm{SI}}P'^{\,2}c^2}.$$

Factor ${}_{\mathrm{SI}}M'\,c^2$:

$${}_{\mathrm{SI}}E' = {}_{\mathrm{SI}}M'\,c^2\,\sqrt{1 + \frac{{}_{\mathrm{SI}}P'^{\,2}}{{}_{\mathrm{SI}}M'^{\,2}\,c^2}}.$$

Using $\sqrt{1+x} = 1 + x/2 + O(x^2)$:

$${}_{\mathrm{SI}}E' = {}_{\mathrm{SI}}M'\,c^2 + \frac{{}_{\mathrm{SI}}P'^{\,2}}{2\,{}_{\mathrm{SI}}M'} + O({}_{\mathrm{SI}}P'^{\,4}).$$

The first term is the rest contribution. The second term is the non-relativistic kinetic term.

#### Operator Substitution

Using the reduced operator correspondences introduced earlier:

$${}_{\mathrm{SI}}E' \to i\,\frac{c}{2\pi}\,\frac{\partial}{\partial t}, \qquad {}_{\mathrm{SI}}P' \to -i\,\frac{c}{2\pi}\,\nabla.$$

Substitute into the kinetic portion:

$$i\,\frac{c}{2\pi}\,\frac{\partial\psi}{\partial t} = {}_{\mathrm{SI}}M'\,c^2\,\psi - \frac{c^2}{8\pi^2\,{}_{\mathrm{SI}}M'}\,\nabla^2\psi.$$

### Removal of Rest Term

Subtracting the constant rest term ${}_{\mathrm{SI}}M'\,c^2$ yields

$$i\,\frac{c}{2\pi}\,\frac{\partial\psi}{\partial t} = -\frac{c^2}{8\pi^2\,{}_{\mathrm{SI}}M'}\,\nabla^2\psi.$$

This is precisely the $\ell_m$-reduced free-particle Schrödinger equation derived previously.

### Conclusion

The Schrödinger equation therefore arises as the low-momentum limit of the reduced relativistic invariant. No additional assumptions are introduced. No geometric structure is invoked. All kilogram dependence remains eliminated.

**Hierarchy Reminder (Schrödinger from the Reduced Invariant)**

- Start from ${}_{\mathrm{SI}}E'^{\,2} = {}_{\mathrm{SI}}P'^{\,2}\,c^2 + {}_{\mathrm{SI}}M'^{\,2}\,c^4$.
- Expand for ${}_{\mathrm{SI}}P'^{\,2} \ll {}_{\mathrm{SI}}M'^{\,2}\,c^2$.
- Apply reduced operator substitutions.
- Subtract rest term.
- Recover $\ell_m$-reduced Schrödinger.

**Interpretation.** Schrödinger is the low-velocity expansion of the kg-free relativistic backbone.

# 10 Relativity Under $\ell_m$-Reduction

## 10.1 Scope

This chapter applies $\ell_m$-reduction to relativistic relations. No reinterpretation of spacetime geometry is introduced. All results follow from the algebraic replacement rules of Chapter 1.

## 10.2 Lorentz Kinematics

The Lorentz factor is defined in SI as

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}.$$

The expression contains no kilogram dependence. Therefore, under $\ell_m$-reduction, $\gamma$ is unchanged. Dimensionless quantities remain invariant.

## 10.3 Minkowski Interval

In SI units, the invariant spacetime interval is

$$s^2 = c^2 t^2 - x^2 - y^2 - z^2.$$

No kilogram appears in this expression. Therefore, the interval remains unchanged under $\ell_m$-reduction. Spacetime geometry is algebraically unaffected.

## 10.4 Four-Momentum

In SI, the four-momentum is defined as

$$p^\mu = \left( \frac{E}{c}, \, \mathbf{p} \right).$$

Apply $\ell_m$-reduction: $E = {}_{\mathrm{SI}}E' \, \ell_m$ and $\mathbf{p} = {}_{\mathrm{SI}}\mathbf{P}' \, \ell_m$. Thus,

$$p^\mu = \ell_m \left( \frac{{}_{\mathrm{SI}}E'}{c}, \, {}_{\mathrm{SI}}\mathbf{P}' \right).$$

Dividing by $\ell_m$ defines the reduced four-momentum:

$${}_{\mathrm{SI}}P'^\mu = \left( \frac{{}_{\mathrm{SI}}E'}{c}, \, {}_{\mathrm{SI}}\mathbf{P}' \right).$$

No kilogram remains.

## 10.5 Relativistic Invariant

In SI:

$$E^2 = p^2 c^2 + m^2 c^4.$$

Substitute $E = {}_{\mathrm{SI}}E' \, \ell_m$, $p = {}_{\mathrm{SI}}P' \, \ell_m$, and $m = {}_{\mathrm{SI}}M' \, \ell_m$. Factor $\ell_m^2$ and cancel:

$${}_{\mathrm{SI}}E'^2 = {}_{\mathrm{SI}}P'^2 \, c^2 + {}_{\mathrm{SI}}M'^2 \, c^4.$$

The algebraic structure is preserved exactly.

### 10.6 Einstein Field Equation Under $\ell_m$-Reduction

In SI form:

$$G_{\mu\nu} = \frac{8\pi G}{c^4}\, T_{\mu\nu}.$$

The Einstein tensor $G_{\mu\nu}$ carries no explicit mass dimension. Kilogram dependence appears in the gravitational constant $G$ and the stress–energy tensor $T_{\mu\nu}$.

Define reduced stress–energy:

$${}_{\mathrm{SI}}T'_{\mu\nu} := \frac{T_{\mu\nu}}{\ell_m}.$$

All kilogram dependence is eliminated.

Define reduced gravitational constant:

$${}_{\mathrm{SI}}G' := G\,\ell_m.$$

*Notation note (revised v0.2).* ${}_{\mathrm{SI}}G'$ is an $\ell_m$-reduced dimensional quantity within Tier 3. The SI left-subscript carries the disambiguation that v0.1 carried verbally: the Arc I corridor operator $\sqrt{G'}$ uses the bare prime and is not algebraically related to ${}_{\mathrm{SI}}G'$. The boundary remains as a matter of substance — the two quantities have different definitions, different roles, and different dimensional values — but is now signaled typographically. See the Notation Convention in the Preface and the LMR Tier 3 Glossary, Notation Collisions Table.

Substitute into the field equation:

$$G_{\mu\nu} = \frac{8\pi\,{}_{\mathrm{SI}}G'}{c^4}\,{}_{\mathrm{SI}}T'_{\mu\nu}.$$

All explicit kilogram dependence has been removed.

**Hierarchy Reminder (Relativity Under $\ell_m$-Reduction)**

**Special Relativity**
- Lorentz factor $\gamma$ unchanged (dimensionless).
- Minkowski interval unchanged.
- Four-momentum: $p^\mu = \ell_m\,{}_{\mathrm{SI}}P'^\mu$.
- Invariant reduces to: ${}_{\mathrm{SI}}E'^2 = {}_{\mathrm{SI}}P'^2\,c^2 + {}_{\mathrm{SI}}M'^2\,c^4$.
- Mass frequency factor: $mc/\hbar \longrightarrow 2\pi\,{}_{\mathrm{SI}}M'$.

**General Relativity**
- Define ${}_{\mathrm{SI}}T'_{\mu\nu} = T_{\mu\nu}/\ell_m$.
- Define ${}_{\mathrm{SI}}G' = G\,\ell_m$.
- Einstein equation becomes: $G_{\mu\nu} = (8\pi\,{}_{\mathrm{SI}}G'/c^4)\,{}_{\mathrm{SI}}T'_{\mu\nu}$.

**Properties**
- All explicit kilogram dependence eliminated.
- Algebraic structure preserved exactly.
- No geometric reinterpretation introduced.
- No structural ontology invoked.

**Conclusion.** $\ell_m$-reduction preserves the algebraic structure of relativistic theory.

**Hierarchy Reminder (Relativity Hierarchy)**

**Tier 1 — Structural Grammar.** No spacetime metric is derived. No curvature dynamics are introduced. No field equations arise at the structural level.
**Tier 2 — Overlay Layer.** Wave and telegrapher forms arise from declared update symmetry. Relativistic equation forms may be admitted only under explicit postulates. No spacetime metric or invariant structure is derived at this tier.
**Tier 3 — Dimensional Correlation.** Lorentz kinematics remain algebraically unchanged. The Minkowski interval is unaffected. The relativistic invariant reduces to ${}_{\mathrm{SI}}E'^{\,2} = {}_{\mathrm{SI}}P'^{\,2}\,c^2 + {}_{\mathrm{SI}}M'^{\,2}\,c^4$. The Einstein field equation may be written in kg-free form. No geometric reinterpretation is introduced.

# 11 Dimensional Pattern Synthesis

**Purpose.** This chapter consolidates patterns revealed by $\ell_m$-reduction. No new equations are introduced. No interpretation beyond dimensional structure is made.

## 11.1 Mass–Length Reciprocity

From Chapter 6:

$$_{\mathrm{SI}}M' = \frac{m}{\ell_m}.$$

From the Compton identity:

$$\lambda_C = \frac{h}{mc} \implies {}_{\mathrm{SI}}M'\,\lambda_C = 1.$$

Thus, reduced mass and characteristic length appear as reciprocal quantities. This reciprocity is dimensional and algebraic. No geometric structure is assumed.

## 11.2 Frequency–Mass Correspondence

From Planck reduction: ${}_{\mathrm{SI}}E' = cf$. From relativistic reduction: ${}_{\mathrm{SI}}E' = {}_{\mathrm{SI}}M'\,c^2$. Equating:

$${}_{\mathrm{SI}}M'\,c^2 = cf \implies f = {}_{\mathrm{SI}}M'\,c.$$

Frequency scales linearly with reduced mass.

## 11.3 Atomic Scaling Pattern

From Chapter 6:

$$r_e \propto \frac{\alpha}{{}_{\mathrm{SI}}M'}, \qquad a_0 \propto \frac{1}{\alpha\,{}_{\mathrm{SI}}M'}, \qquad R_\infty \propto \alpha^2\,{}_{\mathrm{SI}}M'.$$

Observed pattern:

- Length quantities are proportional to $1/{}_{\mathrm{SI}}M'$.
- Frequency quantities are proportional to ${}_{\mathrm{SI}}M'$.
- Dimensionless constants remain unchanged.

## 11.4 Relativistic Backbone

From Chapter 10:

$${}_{\mathrm{SI}}E'^2 = {}_{\mathrm{SI}}P'^2\,c^2 + {}_{\mathrm{SI}}M'^2\,c^4.$$

Mass enters quadratically. Momentum remains linear in ${}_{\mathrm{SI}}M'$. No structural alteration occurs under reduction.

## 11.5 Wave Operator Pattern

Across Schrödinger, Klein–Gordon, and Dirac:

- $\hbar$ disappears.
- $mc/\hbar$ reduces to $2\pi\,{}_{\mathrm{SI}}M'$.
- $m^2c^2/\hbar^2$ reduces to $4\pi^2\,{}_{\mathrm{SI}}M'^2$.

The repeated appearance of $2\pi$ arises from the $\hbar = h/(2\pi)$ convention. No additional dimensional factors remain.

**Hierarchy Reminder (Dimensional Patterns Revealed by $\ell_m$-Reduction)**

- Mass and Compton length appear as reciprocals. This reciprocity is specific to Compton pairing and must not be generalized to arbitrary length scales.
- Frequency scales linearly with ${}_{\mathrm{SI}}M'$.
- Length scales inversely with ${}_{\mathrm{SI}}M'$.
- Relativistic structure is unchanged.
- Quantum equations preserve form after kg elimination.
- All surviving dimensional structure depends only on $c$ and ${}_{\mathrm{SI}}M'$.
- Dimensionless constants remain invariant.

**Conclusion.** $\ell_m$-reduction reorganizes SI expressions without altering their predictive structure.

# The Correlation and the Boundary

## Dimensional Correlation Summary

Companion Volume II has examined the dimensional correlation layer of the LMR program. Its purpose is not to extend the predynamical codex, but to show how standard SI expressions may be rewritten after isolating the kilogram carrier through the bridge quantity

$$\ell_m = \frac{h}{c}.$$

Under $\ell_m$-reduction, SI mass is represented by

$${}_{\mathrm{SI}}M' = \frac{m}{\ell_m} = \frac{1}{\lambda_C},$$

so that mass-bearing expressions may be written in inverse-length form while preserving algebraic structure. The reduction is exact and reversible. No physical content is removed, and no new physical mechanism is introduced.

Across the examples considered, the same pattern recurs: explicit kilogram dependence is isolated into $\ell_m$, while the reduced expressions retain their original algebraic relationships in terms of length, time, $c$, dimensionless factors, and reduced quantities such as ${}_{\mathrm{SI}}M'$.

This convergence does not establish structural ontology. It does not derive the Tier 1 codex and does not select among Tier 2 overlays. It shows only that familiar SI equations admit a common kilogram-free dimensional face.

The result is therefore a dimensional correlation result. Companion Volume II clarifies how the standard SI representation may be aligned with the LMR grammar without modifying either the empirical equations or the predynamical codex.

## Document Status and Boundary

This document is a non-normative companion to the LMR predynamical codex. It provides dimensional correlation and representational alignment of standard SI expressions under $\ell_m$-reduction. It does not introduce structural primitives, corridor operators, or admissibility conditions, and does not modify the codex grammar. All structural authority remains with the Arc 1 papers.

Dimensional reduction preserves algebraic structure but does not establish structural necessity. Interpretive conclusions do not arise from dimensional correspondence alone.

The present document is dependent on the codex and should be read only after the foundational sequence.

Readers are directed to the Arc 1 papers for governing definitions, or to the Concepts and Reference sections for structured navigation. The LMR Tier 3 Glossary is the authoritative reference for the notational conventions used in this volume.

## Companion Series Relation

Companion Volume II and Companion Volume I form a paired non-normative series within the LMR program.

Volume II performs dimensional correlation of standard SI expressions under $\ell_m$-reduction. Volume I introduces declared representational overlays operating on structural cadence scales.

The two volumes are formally independent but algebraically compatible. Neither volume modifies structural grammar, and neither establishes ontological claims.

Together, they provide dimensional and representational context to the predynamical codex without altering its structure.